Paper Type: Original Article

Distance Measure of Picture Fuzzy Sets and Its Application in Multi-Criteria Decision Making Problem

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Citation:

Abstract

Distance measurement is an important technique for determining the similarity of two Picture Fuzzy Sets (PFSs). The article presents a novel idea of distance measure for PFSs, which is then applied to solve Multi-Criteria Decision-Making (MCDM) problems. By using the distance measure of the PFSs, the TOPSIS approach is applied to solve MCDM problems. The score function is employed to determine the best alternatives for both Picture Fuzzy Positive Ideal Solution (PFPIS) and Picture Fuzzy Negative Ideal Solutions (PFNIS). The options are ranked using the revised index technique and the relative closeness coefficient method. An illustrative numerical example is provided to demonstrate the feasibility and practicality of the proposed distance measure for PFSs.

Keywords: Intuitionistic fuzzy sets, Picture fuzzy sets, Similarity measure, Score function, Revised index method.

1 | Introduction

In our daily lives, uncertainty is unavoidable. We do not build this universe on assumptions or precise measures. It is impossible to make decisions in advance. We face a significant problem in dealing with errors in decision-making. In 1965, Zadeh [1] proposed Fuzzy Sets (FSs) as a way to deal with ambiguity in real-world problems. Every component of the conceptual universe undergoes a renumbering from the unit range [0, 1] to represent the degree of belonging to the set under study. We designate a number from the unit range [0, 1] for each element of the discursive multiverse to signify the degree of sense of esteem and self-actualization in the set under study. FSs are a subclass of set theory that allows for states that are midway between completeness and nothingness. A FS uses a membership function to express the extent to which an element belongs to a class. Membership has a value ranging from 0 to 1. The grade 0 indicates that the element

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is not a member of the class; the grade 1 indicates that it is; and additional considerations indicate the level of participation.

When it comes to decision-making, assigning a membership value is not always adequate. FSs can only express vagueness; they can't handle the hesitation inherent in human thinking. Atanassov [2] developed Intuitionistic Fuzzy Sets (IFSs), important generalizations of FSs, to define hesitations more clearly. This approach uses the degree of membership and non-membership to model vagueness and impression, while the sum of membership and non-membership is less than or equal to 1. The main contribution of IFSs is their ability to deal with hesitancy that may exist due to imprecise information. Because of its ability to address uncertainty, it has achieved success in a variety of fields. IFS has many applications throughout several disciplines, incorporating medical diagnosis [3], pattern recognition [4], [5], decision-making problems [6], [7], image processing, etc. Researchers found several results in the field of uncertainty in [8–10]. In Neutrosophic Sets (NS), Edalatpanah [11] introduced a new concept called Neutrosophic Structured Element (NSE). Based on this concept, NS's operational laws, score function, and some aggregation operators should be defined.

Despite having numerous successful applications, IFSs cannot manage contradictory information in the actual world. For instance, while voting on a question, the four possible outcomes are: "vote for", "abstain", "vote against", and "refuse to vote". Cuong [10] suggested the use of a Picture Fuzzy Set (PFS) to address this kind of problem. The composition of PFS includes the membership grade, the neutral membership grade, and the non-membership grade. The PFS, which also serves for clustering, fuzzy inference, pattern recognition, and decision-making, effectively solved the voting problem.

The similarity measure is a crucial tool for assessing the degree of similarity between two intuitive FSs. Different parameters have led to the development of various similarity measures for IFSs. Li and Chen [12] utilized the medians of two intervals to compute the similarity measure of IFSs. Liang and Shi [13] further refined and enhanced these measures through numerical evaluations, offering improvements over Li and Chen's methods. Xu and Chen [14] conducted a comprehensive analysis and comparison of distance and similarity measurements among IFSs. More recently, Hwang and Yang [15] introduced a novel approach for distance measures for IFSs by defining upper, middle, and lower FSs, demonstrating enhancements over existing methods. Szmidt et al. [16] have constructed similarity measures based on hamming distance measures and then applied them to a multi-attribute decision-making problem. Hung et al. [17] constructed a similarity measure using Hausdorff distance. Ye [18] proposed a cosine measure for IFSs using the cosine function. To provide an overview of the prior proposed similarity, Baccour et al. [19] summarised the similarity measures and pointed out that each similarity measure has drawbacks. Chaira et al. [20] discussed some results on new distance measures of IFSs. Several important results in the fields of uncertainty, like IFSs, Pythagorean FSs, and Pythagorean FSs, can be found in [21–26].

This paper aims to propose a similarity measure for PFSs that can more accurately gauge the degree of similarity between PFs. The primary objective of this paper is to propose a distance measure for PFSs that more accurately gauges the degree of similarity between them. The proposed distance measure is a combination of Hamming distances, Hausdroff distances, and Euclidean distances. In this method, membership grade, neutral ship grade, and non-membership grade are considered parameters. The discussion also includes the normalized version of the proposed distance measure. Given the existing limitations in the ranking function and operation rules for PFSs, the proposed distance measure aims to address these shortcomings. Consequently, this paper will devise a new decision-making method leveraging the proposed similarity measure for Multi-Attribute Decision Making (MADM) in a Picture fuzzy environment, assuming known attribute weight information.

Here's the outline for the remainder of the paper. Section 2 gives a brief synopsis of IFSs and PFSs, including their definition and certain operations on them. In Section 3, we explore how to use the TOPSIS approach to solve Multi-Criteria Decision-Making (MCDM) issues in a Picture-fuzzy situation by measuring distances between features. In Section 4, we will go over the suggested approach using numerical examples. Section 5 concludes the study.

2 | Preliminaries

Here, we elucidated the fundamental concepts of FSs, IFSs, and PFSs. We also explained some basic operations, such as the union and intersection of two sets, the complement of a set, and the score function defined on these sets.

Traditional set theory evaluates membership within a set using binary values of 0 and 1, signifying whether an element is part of the set or not. In real-life situations, this is practically not possible. To overcome this challenge, we expanded the classical set theory to include the innovative FS concept, in which we evaluated the members using a function ranging from the universal set to the membership grade.

Definition 1 ([1]). A FS F defined on the universe Ω is defined in terms of membership function $\mu_F : F \rightarrow [0,1]$ associated with elements of the set F and it is expressed as

$$F = \{(\xi, \mu_F(\xi)) : \xi \in \Omega\}.$$

in this context, $\mu_F(\xi)$ represents the extent to which the element ξ belongs to the set F.

The IFS, which further extends the FS theory, evaluates members using two functions from the universal set: membership grade and non-membership grade, respectively.

Definition 2 ([2]). An IFS I defined on universe Ω is defined in terms of membership grade $\mu_I : I \rightarrow [0,1]$ and non-membership grade $\sigma_I : I \rightarrow [0,1]$ associated with elements of the set I and it is expressed as

$$I = \{(\xi, \mu_I(\xi), \sigma_I(\xi)) : \xi \in \Omega\}.$$

Given that $0 \le \mu_I(\xi) + \sigma_I(\xi) \le 1$, where $\mu_I(\xi)$ and $\sigma_I(\xi)$ denote the membership grade and non-membership grade of ξ to I respectively.

The IFS theory is further extended to the PFS in which the members are assessed in terms of three functions, each from the universal set to [0,1], which are known as membership grade, neutralship grade, and non-membership grade, respectively. Using this concept, Cuong [27] introduced the notion of a PFS.

Definition 3. A PFS P defined on universe Ω is defined in terms of membership grade $\mu_P : P \rightarrow [0,1]$, neutralship grade $\rho_P : P \rightarrow [0,1]$ and non-membership grade $\sigma_P : P \rightarrow [0,1]$ associated with elements of the set P, and it is expressed as

$$P = \{(\xi, \mu_P(\xi), \rho_P(\xi), \sigma_P(\xi)) : \xi \in \Omega\}.$$

Given that $0 \le \mu_P(\xi) + \rho_P(\xi) + \sigma_P(\xi) \le 1$, where $\mu_P(\xi)$, $\rho_P(\xi)$ and $\sigma_P(\xi)$ denote the degree of membership, degree of neutalship and degree of non-membership of ξ to S respectively.

Basically, $1 - (\mu_P(\xi) + \rho_P(\xi) + \sigma_P(\xi))$ could be called the degree of refusal membership of ξ to Ω.

2.1 | Some Basic Operations on PFSs

Let $PFS(\Omega)$ denotes the set of all PFSs defined on the universe Ω.

Definition 4. Let $P = \{(\xi, \mu_P(\xi), \rho_P(\xi), \sigma_P(\xi))\}$ and $Q = \{(\xi, \mu_Q(\xi), \rho_Q(\xi), \sigma_Q(\xi))\}$ be any two PFS defined on Ω.

We defined the subset-hood and equality as

$$P \subseteq Q \Leftrightarrow (\text{For all } \xi \in \Omega)(\mu_P(\xi) \le \mu_Q(\xi) \text{ and } \rho_P(\xi) \le \rho_Q(\xi) \text{ and } \sigma_P(\xi) \ge (\sigma_Q(\xi))).$$

$$P = Q \Leftrightarrow (\text{For all } \xi \in \Omega)(P \subseteq Q \text{ and } Q \subseteq P).$$

We defined the union and intersection of P and Q as

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$P \cup Q = \{(\xi, (\max(\mu_P(\xi), \mu_Q(\xi)), \max(\rho_P(\xi), \rho_Q(\xi)), \min(\sigma_P(\xi), \sigma_Q(\xi))) : \xi \in \Omega\}.$

$P \cap Q = \{(\xi, (\min(\mu_P(\xi), \mu_Q(\xi)), \min(\rho_P(\xi), \rho_Q(\xi)), \max(\sigma_P(\xi), \sigma_Q(\xi))) : \xi \in \Omega\}.$

The Complement of P is defined as

$P^{'} = \{(\xi, \sigma_P(\xi), \rho_P(\xi), \mu_P(\xi)) : \xi \in \Omega\}.$

Definition 5. The score function the PFSs P is defined as

$Score(P) = \mu_P^2(x) + \rho_P^2(x) - \sigma_P^2(x).$

Definition 6. A function $d : PFS(\Omega) \rightarrow [0, \infty)$ is defined as a distance measure between two PFSs A and B if it satisfies the following criteria:

   I. $d(A, B) \geq 0.$

  II. $d(A, B) = 0$ iff $A = B.$

 III. $d(A, B) = d(B, A)$ for all $A, B \in PFS(\Omega).$

 IV. $d(A, C) \leq d(A, B) + d(B, C),$ for all $A, B, C \in PFS(\Omega).$

2.2 | Distance Measure of PFSs

In this section, we propose new distance measures for PFSs defined on the universe of discourse X by taking into account all three parameters: the degree of positive membership $(\mu(x))$, the degree of neutralship $(\rho(x))$, and the degree of non-membership $(\sigma(x))$.

Definition 7. Let $\Omega = \{\xi_1, \xi_2, \xi_3, \ldots \xi_n\}$ be the universal set. Consider any two PFSs $S_1 = \{(\xi, \mu_{S_1}(\xi), \rho_{S_1}(\xi), \sigma_{S_1}(\xi))\}, S_2 = \{(\xi, \mu_{S_2}(\xi), \rho_{S_2}(\xi), \sigma_{S_2}(\xi))\} \in PFS(\Omega);$ the distance measures between PFSs are as follows.

$$D_H(S_1, S_2) = \frac{1}{3n} \sum_{i=1}^{n} \left[|\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i)| + |\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i)| + |\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i)| \right].$$

$$D_E(S_1, S_2) = \frac{1}{n} \left\{ \sum_{i=1}^{n} \left[(\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i))^2 + (\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i))^2 + (\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i))^2 \right] \right\}^{\frac{1}{2}}.$$

$$D_H^m(S_1, S_2) = \frac{1}{n} \left\{ \sum_{i=1}^{n} \max \left[|\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i)|, |\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i)|, |\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i)| \right] \right\}.$$

$$D_E^m(S_1, S_2) = \frac{1}{n} \left\{ \sum_{i=1}^{n} \max \left[|\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i)|^2, |\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i)|^2, |\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i)|^2 \right] \right\}^{\frac{1}{2}}.$$

Example 1. Let us consider three PFSs S_1, S_2 and S_3, where

$S_1 = \{(0.302, 0.405, 0.202), (0.902, 0.001, 0.012), (0.412, 0.312, 0.015)\}.$

$S_2 = \{(0.205, 0.309, 0.401), (0.304, 0.409, 0.102), (0.309, 0.501, 0.005)\}.$

$S_3 = \{(0.404, 0.302, 0.201), (0.301, 0.205, 0.001), (0.401, 0.301, 0.101)\}.$

Then the different distances are

$D_H(S_1, S_2) = 0.19888, \quad D_H(S_2, S_3) = 0.1224, \quad D_H(S_1, S_3) = 0.12555.$

$D_E(S_1, S_2) = 0.2660, \quad D_E(S_2, S_3) = 0.1449, \quad D_E(S_1, S_3) = 0.2189.$

$D_H^m(S_1,S_2) = 0.8533, \ D_H^m(S_2,S_3) = 0.7213, \ D_H^m(S_1,S_3) = 0.4707.$

$D_E^m(S_1,S_2) = 0.0731, \ D_E^m(S_2,S_3) = 0.0684, \ D_E^m(S_1,S_3) = 0.0387.$

Some results on distance measures between PFSs are discussed below:

Lemma 1. Let Ω be the universe of discourse, then the distance measures $D_H(S_1,S_2), \ D_E(S_1,S_2), \ D^m{}_H(S_1,S_2)$ and $D^m{}_E(S_1,S_2)$ are metric.

Proof: we give the proof only for $D_H(S_1,S_2),$ and the remaining can be proved in similar ways.

Let us consider two PFSs $S_1 = \{(\xi, \mu_{S_1}(\xi), \rho_{S_1}(\xi), \sigma_{S_1}(\xi))\},$ and $S_2 = \{(\xi, \mu_{S_2}(\xi), \rho_{S_2}(\xi), \sigma_{S_2}(\xi))\} \in PFS(\Omega)$ of the universe discourse $\Omega = \{\xi_1, \xi_2, \xi_3, ... \xi_n\},$ then

   I. $D_H(S_1,S_2) \geq 0,$ because of absolute value properties.

   II. If $S_1 = S_2,$ then $\mu_{S_1}(\xi) = \mu_{S_2}(\xi), \rho_{S_1}(\xi) = \rho_{S_2}(\xi), \sigma_{S_1}(\xi) = \sigma_{S_2}(\xi)$ for each $\xi \in \Omega$ and hence
$$D_H(S_1,S_2) = D_H(S_2,S_1).$$

     Conversely, let $D_H(S_1,S_2) = 0,$ then for each $\xi \in \Omega,$ we have $\left|\mu_{S_1}(\xi) - \mu_{S_2}(\xi)\right| = 0, \ \left|\rho_{S_1}(\xi) = \rho_{S_2}(\xi)\right| = 0,$ $\left|\sigma_{S_1}(\xi) = \sigma_{S_2}(\xi)\right| = 0$ and so, we conclude that $S_1 = S_2.$

   III. The symmetry of the property can be shown below:

$$D_H(S_1,S_2) = \frac{1}{3n} \sum_{i=1}^{n} \left[|\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i)| + |\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i)| + |\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i)| \right]$$

$$= \frac{1}{3n} \sum_{i=1}^{n} \left[|\mu_{S_2}(\xi_i) - \mu_{S_1}(\xi_i)| + |\rho_{S_2}(\xi_i) - \rho_{S_1}(\xi_i)| + |\sigma_{S_2}(\xi_i) - \sigma_{S_1}(\xi_i)| \right]$$

$$= D_H(S_2,S_1).$$

   IV. For triangular inequality, we consider three PFSs $S_1 = \{(\xi, \mu_{S_1}(\xi), \rho_{S_1}(\xi), \sigma_{S_1}(\xi))\},$
$S_2 = \{(\xi, \mu_{S_2}(\xi), \rho_{S_2}(\xi), \sigma_{S_2}(\xi))\}$ and $S_3 = \{(\xi, \mu_{S_3}(\xi), \rho_{S_3}(\xi), \sigma_{S_3}(\xi))\} \in PFS(\Omega)$ of universe discourse $\Omega = \{\xi_1, \xi_2, \xi_3, ... \xi_n\},,$ and we have to prove $D_H(S_1,S_3) \leq D_H(S_1,S_2) + D_H(S_2,S_3).$

Now,

$$D_H(S_1,S_3) = \frac{1}{3n} \sum_{i=1}^{n} \left[|\mu_{S_1}(\xi_i) - \mu_{S_3}(\xi_i)| + |\rho_{S_1}(\xi_i) - \rho_{S_3}(\xi_i)| + |\sigma_{S_1}(\xi_i) - \sigma_{S_3}(\xi_i)| \right]$$

$$= \frac{1}{3n} \sum_{i=1}^{n} \left[\left|\left(\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i)\right) + \left(\mu_{S_2}(\xi_i) - \mu_{S_3}(\xi_i)\right)\right| + \left|\left(\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i)\right) + \left(\rho_{S_2}(\xi_i) - \rho_{S_3}(\xi_i)\right)\right| \right.$$

$$\left. + \left|\left(\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i)\right) + \left(\sigma_{S_2}(\xi_i) - \sigma_{S_3}(\xi_i)\right)\right| \right]$$

$$\leq \frac{1}{3n} \sum_{i=1}^{n} \left[\left|\left(\mu_{S_1}(\xi_i) - \mu_{S_2}(\xi_i)\right)\right| + \left|\left(\rho_{S_1}(\xi_i) - \rho_{S_2}(\xi_i)\right)\right| + \left|\left(\sigma_{S_1}(\xi_i) - \sigma_{S_2}(\xi_i)\right)\right| \right]$$

$$+ \frac{1}{3n} \sum_{i=1}^{n} \left[\left|\left(\mu_{S_2}(\xi_i) - \mu_{S_3}(\xi_i)\right)\right| + \left|\left(\rho_{S_2}(\xi_i) - \rho_{S_3}(\xi_i)\right)\right| + \left|\left(\sigma_{S_2}(\xi_i) - \sigma_{S_3}(\xi_i)\right)\right| \right]$$

$$= D_H(S_1,S_2) + D_H(S_2,S_3).$$

Therefore, $D_H(S_1,S_3) \leq D_H(S_1,S_2) + D_H(S_2,S_3).$

Hence, $D_H(S_1,S_2)$ is a metric.

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Lemma 2. For any two PFSs $S_1 = \{(\xi, \mu_{S_1}(\xi), \rho_{S_1}(\xi), \sigma_{S_1}(\xi))\}$, $S_2 = \{(\xi, \mu_{S_2}(\xi), \rho_{S_2}(\xi), \sigma_{S_2}(\xi))\} \in PFS(\Omega)$, of the universe discourse $\Omega = \{\xi_1, \xi_2, \xi_3, \dots \xi_n\}$, the following inequalities hold:

$$D_E^m(S_1, S_2) \le D_H(S_1, S_2) \le D_E(S_1, S_2) \le D_H^m(S_1, S_2).$$

Proof: proof of the above result is straightforward.

3 | Proposed Method

In this section, we introduce MCDM problems in a picture-fuzzy environment. We introduce the Picture fuzzy TOPSIS system to address MCDM challenges in a fuzzy environment. The TOPSIS approach's core principle is to identify the most desirable alternative, which should be not only the shortest distance from the positive ideal solution but also the greatest distance from the negative ideal solution.

An MCDM problem is expressed as a decision matrix, the elements of which are the assessment values of all alternatives for every criterion.

Let $C = \{C_1, C_2, \cdots, C_n\}$, with $n \ge 2$, represent a set of criteria and $V = \{\xi_1, \xi_2, \dots, \xi_m\}$, with $m \ge 2$, represent a set of alternatives. Let $w = (w_1, w_2, \dots, w_n)^T$ be the weight vector for criterion, where $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i = 1$.

The Picture fuzzy numbers $\langle \mu_{ij}, \rho_{ij}, \sigma_{ij} \rangle$ represent the assessment value of the i-th alternative for the j-th criterion, denoted as $C_j(\xi_i) = \langle \mu_{ij}, \rho_{ij}, \sigma_{ij} \rangle$. Let $R = (C_j(\xi_i))_{m \times n}$ denote the Picture fuzzy decision matrix,

$$R = \begin{bmatrix} \langle \mu_{11}, \rho_{11}, \sigma_{11} \rangle & \langle \mu_{12}, \rho_{12}, \sigma_{12} \rangle & \cdots & \langle \mu_{1n}, \rho_{1n}, \sigma_{1n} \rangle \\ \langle \mu_{21}, \rho_{21}, \sigma_{21} \rangle & \langle \mu_{22}, \rho_{22}, \sigma_{22} \rangle & \cdots & \langle \mu_{2n}, \rho_{2n}, \sigma_{2n} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle \mu_{m1}, \rho_{m1}, \sigma_{m1} \rangle & \langle \mu_{m2}, \rho_{m2}, \sigma_{m2} \rangle & \cdots & \langle \mu_{mn}, \rho_{mn}, \sigma_{mn} \rangle \end{bmatrix}.$$

where the degree of positive membership $\mu_{ij}(x)$, degree of neutral membership $\rho_{ij}(x)$ and the degree of non-membership $\sigma_{ij}(x)$.

3.1 | Solution Procedure of the Proposed Method

First, classify the criteria into two categories: one benefit criteria and the other cost criteria. Let J_1 represent the set of benefit criteria and J_2 denote the set of cost criteria. PFPIS and PFNIS are determined using a scoring function. Denoting ξ^+ and ξ^- as the PFPIS and PFNIS respectively, are given by

$$\xi^+ = \{C_j, \max(S(C_j(\xi_i))) \mid j = 1, 2, \dots, n\}. \tag{1}$$

$$\xi^- = \{C_j, \min(S(C_j(\xi_i))) \mid j = 1, 2, \dots, n\}. \tag{2}$$

where $S(C_j(\xi_i))$ is the score function for the picture fuzzy numbers.

Next, we compute the distance from each alternative to the PFPIS $D(\xi_i, \xi^+)$ and PFNIS $D(\xi_i, \xi^-)$. Subsequently, we derive the weighted distance of alternative ξ_i from PFPIS ξ^+ based on a similarity measure:

$$D(\xi_i, \xi^+) = \sum_{j=1}^{n} w_j D(C_j(\xi_i), C_j(\xi^+)), \tag{3}$$

where $i = 1, 2, \dots, n$.

After that, based on the TOPSIS principle, the smaller $D(\xi_i, \xi^+)$, the better the alternative ξ_i and the minimum value of $D(\xi_i, \xi^+)$ depicted as $D_{min}(\xi_i, \xi^+)$.

Similarly, the weighted distance of alternative ξ_i from PFNIS ξ^- is calculated as follows:

$$D(\xi_i, \xi^-) = \sum_{j=1}^{n} w_j D(C_j(\xi_i), C_j(\xi^-)), \tag{4}$$

where $i = 1, 2, \ldots, n$.

Based on the TOPSIS principle, the greater $D(\xi_i, \xi^-)$, the better the alternative ξ_i and the maximum value of $D(\xi_i, \xi^-)$ denoted as $D_{max}(\xi_i, \xi^-)$..

To rank the attributes, calculate the relative closeness coefficient ξ_i with respect to PFPIS (ξ^+) and PFNIS (ξ^-). The formula for $RC(\xi_i)$ is as follows:

$$RC(\xi_i) = \frac{D(\xi_i, \xi^-)}{D(\xi_i, \xi^+) + D(\xi_i, \xi^-)}. \tag{5}$$

The optimal solution minimizes the distance from the positive ideal solution and maximizes the distance from the negative ideal solution.

Therefore, we employ the revised index, denoted by $\eta(\xi_i)$, to rank attributes. The formula for the revised index is expressed as follows:

$$\eta(\xi_i) = \frac{D(\xi_i, \xi^-)}{D_{max}(\xi_i, \xi^-)} - \frac{D(\xi_i, \xi^+)}{D_{min}(\xi_i, \xi^-)}. \tag{6}$$

Based on $RC(\xi_i)$ or $\eta(\xi_i)$, we rank the alternatives ξ_i to identify the optimal solution according to the maximum value of $RC(\xi_i)$ or $\eta(\xi_i)$.

3.2|Algorithm for Proposed Method

TOPSIS method is effectively used to deal with the MCDM problems with picture fuzzy information using several distance measurement methods. The algorithm involves the following steps:

Step 1. For the MCDM problem with picture fuzzy data, construct the decision matrix $R = (C_j(\xi_i))_{m \times n}$, where the elements $C_j(\xi_i)$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ are the assessments of alternative ξ_i with respect to the criterion C_j.

Step 2. Utilize the score function to determine the Picture fuzzy positive ideal solution (ξ^+) and the Picture fuzzy negative ideal solution (ξ^-).

Step 3. Use the distance formula to calculate the weighted distances of each alternative ξ_i from the Picture fuzzy PIS (ξ^+) and the Picture fuzzy NIS (ξ^-).

Step 4. *Eq. (5)* and *Eq. (6)* to calculate relative closeness $RC(\xi_i)$ and the revised closeness $\eta(\xi_i)$ of the alternative ξ_i.

Step 5. Rank the alternatives and select the best one(s) according to the decreasing relative closeness $RC(\xi_i)$ and revised closeness $\eta(\xi_i)$ obtained from *Step 4*.

The bigger the $RC(\xi_i)$ or, $\eta(\xi_i)$ the more desirable the ξ_i, $(i = 1, 2, \ldots, m)$ will be.

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4 | Illustrative Example

The exploration of MADM holds considerable importance in modern decision science and finds extensive application across various industries, such as management, medicine, and the economy. MCDM is a process that involves selecting the best option from a list of feasible options. In this section, we'll apply our distance measure to a MCDM process.

Example 2. Contemplate a manufacturing corporation in search of a suitable site to establish a new factory. Let us make the assumption that there are five alternative places denoted as ξ_1, ξ_2, ξ_3, ξ_4 and ξ_5. In order to select the final location, there are four criteria: labor cost (C_1), availability of material (C_2), expansion possibility (C_3), and availability of market (C_4) for establishment of a new factory.

The given Picture fuzzy decision matrix is as follows:

	C_1	C_2	C_3	C_4
ξ_1	(0.5,.01,0.4)	(0.7,0.1,0.1)	(0.5,0.2,0.3)	(0.4,0.2,0.3)
ξ_2	(0.6,0.1,0.3)	(0.4,0.2,0.3)	(0.4,0.1,0.4)	(0.6,0.2,0.2)
ξ_3	(0.4,0.2,0.3)	(0.3,0.1,0.5)	(0.5,0.1,0.2)	(0.5,0.2,0.2)
ξ_4	(0.4,0.1,0.4)	(0.4,0.3,0.2)	(0.4,0.2,0.3)	(0.4,0.1,0.4)
ξ_5	(0.6,0.2,0.2)	(0.3,0.3,0.2)	(0.6,0.1,0.2)	(0.3,0.1,0.4)

Each entry represents a Picture fuzzy number in the format (μ,ρ,σ), where μ denotes the degree of membership, ρ denotes the degree of neutrality, and σ denotes the non-membership for the respective criterion and alternative.

The benefit criteria are expansion possibility (C_2), availability of material (C_3), and availability of market (C_4), denoted by $J_1 = \{C_2, C_3, C_4\}$, while the cost criterion is the cost of labor (C_1), represented by $J_2 = \{C_1\}$.

Using the score function, we determine the PFPIS (ξ^+) and PFNIS (ξ^-) as follows:

$\xi^+ = \{(0.4,0.1,0.4),(0.7,0.1,0.1),(0.6,0.1,0.2),(0.6,0.2,0.2)\}$.

$\xi^- = \{(0.6,0.2,0.2),(0.3,0.1,0.5),(0.4,0.1,0.4),(0.3,0.1,0.4)\}$.

Let $(0.2, 0.25, 0.25, 0.3)$ be the weight vector corresponding to (C_1, C_2, C_3, C_4). We now proceed to calculate the weighted distance between ξ_i and ξ^+ and between ξ_i and ξ^-.

Let $D(\xi_i, \xi^+)$ represent the weighted distance between ξ_i and ξ^+. We calculate $D(\xi_i, \xi^+)$ using the distance measures discussed above separately. In the following, we employ different distance measurement methods to assess the weighted Picture fuzzy distance of each alternative ξ_i from PFPIS and PFNIS.

Table 1. $D_H \& D_E$ distance measurement method.

Measurement →	D_H Method				D_E Method			
Attribute ↓	$D(\xi_i,\xi^+)$	$D(\xi_i,\xi^-)$	$RC(\xi_i)$	$\eta(\xi_i)$	$D(\xi_i,\xi^+)$	$D(\xi_i,\xi^-)$	$RC(\xi_i)$	$\eta(\xi_i)$
ξ_1	0.0154	0.0370	0.7063	0.0000	0.0325	0.0654	0.6677	0.0000
ξ_2	0.0258	0.0267	0.5079	-0.9566	0.0522	0.0433	0.4533	-0.9413
ξ_3	0.0245	0.0237	0.4913	-0.9541	0.2247	0.0345	0.3809	-1.1956
ξ_4	0.0333	0.0275	0.4520	-1.4205	0.0611	0.0523	0.4611	-1.0779
ξ_5	0.0379	0.0187	0.3309	-1.9538	0.0717	0.4021	0.3593	-1.5858

Table 2. D_H^m & D_E^m **Distance measurement method.**

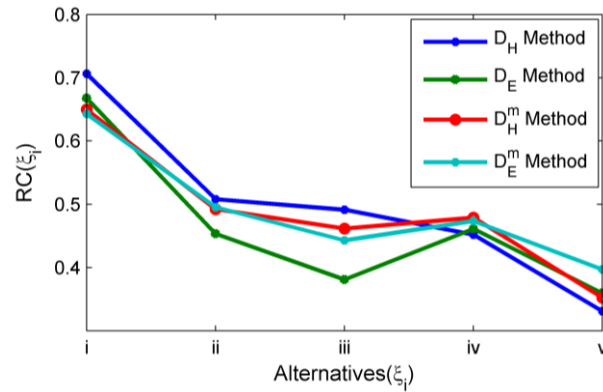| Measurement →	D_H^m Method				D_E^m Method			
Attribute ↓	$D(\xi_i,\xi^+)$	$D(\xi_i,\xi^-)$	$RC(\xi_i)$	$\eta(\xi_i)$	$D(\xi_i,\xi^+)$	$D(\xi_i,\xi^-)$	$RC(\xi_i)$	$\eta(\xi_i)$
ξ_1	0.0262	0.0487	0.6500	0.0000	0.0321	0.0578	0.6429	0.0000
ξ_2	0.0412	0.0400	0.4923	-0.7509	0.0503	0.0493	0.4952	-0.7129
ξ_3	0.0437	0.0375	0.4615	-0.8974	0.0544	0.0433	0.4428	-0.9478
ξ_4	0.462	0.0425	0.4788	-0.8901	0.0527	0.0474	0.4735	-0.8219
ξ_5	0.0575	0.0312	0.3521	-1.5494	0.6846	0.0450	0.3969	-1.3525

Fig. 1. Ranking of alternatives using relative closeness method.

**Table 3. Ranking of the alternatives using
relative closeness method.**

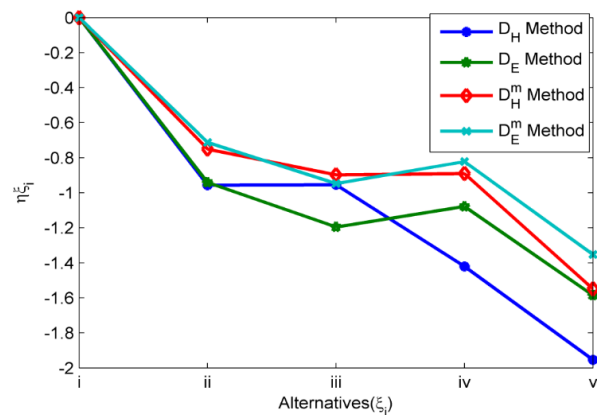| Measurement	Ranking of Alternative
D_H method	$\xi_1 \succ \xi_2 \succ \xi_3 \succ \xi_4 \succ \xi_5$
D_E method	$\xi_1 \succ \xi_4 \succ \xi_2 \succ \xi_3 \succ \xi_5$
D_H^m method	$\xi_1 \succ \xi_2 \succ \xi_4 \succ \xi_3 \succ \xi_5$
D_E^m method	$\xi_1 \succ \xi_2 \succ \xi_4 \succ \xi_3 \succ \xi_5$

Fig. 2. Ranking of alternatives using revised index method

Table 4. Ranking of the alternatives using revised index method.

Measurement	Ranking of alternative
D_H method	$\xi_1 \succ \xi_3 \succ \xi_2 \succ \xi_4 \succ \xi_5$
D_E method	$\xi_1 \succ \xi_2 \succ \xi_4 \succ \xi_3 \succ \xi_5$
D_H^m method	$\xi_1 \succ \xi_2 \succ \xi_4 \succ \xi_3 \succ \xi_5$
D_E^m method	$\xi_1 \succ \xi_2 \succ \xi_4 \succ \xi_3 \succ \xi_5$

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These results illustrate the variations in rankings obtained through different distance measurement methods, highlighting the importance of considering multiple approaches to evaluate alternatives effectively.

5 | Conclusion

In this work, four distance measures for PFSs are introduced. One key benefit of this approach lies in its ability to capture the significance of degrees of membership, neutrality, and non-membership for decision-makers. Additionally, it offers a more comprehensive depiction of the decision-making process, allowing decision-makers to explore numerous scenarios based on their interests by employing different types of similarity measures within the Picture fuzzy environment. The combination of similarity distance measurement and the TOPSIS method with Picture fuzzy data presents a significant opportunity for success in MCDM problems, enabling the ordering of alternatives through relative closeness and the revised index method.

Future research anticipates developing dissimilarity distance measurement methods in various environments, including Intuitionistic fuzzy, Fermatean fuzzy, and Picture fuzzy, among others. This technique has potential, particularly in fields such as engineering, supply chain management and logistics, manufacturing systems, environmental management, business and marketing, water resource management, and human resources.

Acknowledgments

Conflicts of Interest

The authors declare that there are no competing interests.

Funding

Authors contributions

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

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